LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.C.A. DEGREE EXAMINATION – **COMPUTER APPLICATION**

FIRST SEMESTER – APRIL 2010

MT 1902 - MATHEMATICS FOR COMPUTER APPLICATIONS

Date & Time: 30/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

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Part A (Answer ALL questions) **2 x 10 = 20**

1. Define Lattice homomorphism between two lattices.
2. With usual notations prove that (i) $a * a = a$ (ii) $a * b = b * a$.
3. Define context free grammar.
4. What is the difference between deterministic finite automata and non-deterministic finite automata?
5. Let G = (N, T, P, S), where N = {S}, T = {a}, P: {S → SS, S → a}. Check whether G is ambiguous or unambiguous.
6. Give a deterministic finite automata accepting the set of all strings over {0, 1} containing 3 consecutive 0's.
7. If R and S be two relations defined by $R = \left\{\frac{\langle x, 2x\rangle}{x \epsilon I}\right\}$ and $S = \left\{\frac{\langle x, 7x\rangle}{x \epsilon I}\right\}$, then find R∘S, R∘R and $R \circ S \circ R$.
8. Let $X = \{1,2,3,4\}$ and $R = \{(1,1), (1,4)(4,1)(4,4), (2,2)(2,3), (3,2), (3,3)\}$. Write the matrix of of R and sketch its graph.
9. Define ring with an example.
10. State Kuratowski's theorem.

Part B (Answer ALL questions) **5 x 8 = 40**

11. (a) Show that De Morgan's laws given by $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$ hold in a complemented, distributive lattice.

(OR)

(b) Let $\langle L, \leq \rangle$ be a lattice. For any $a, b, c \in L,$ prove the following distributive inequalities: $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$ and $a * (b \oplus c) \geq (a * b) \oplus (a * c)$.

12. (a) Show that L(G) = $\left\{\frac{a^n b^m a^m b^n}{n}, m \geq 1\right\}$ is accepted by the grammar G = (N, T, P, S) where N = {S,A} T = {a, b}, P consists of the following productions: S → aSA, S → aZA, Z → bZB, Z → bB, BA → AB, AB → Ab, bB → bb, bA→ ba.

(OR)

(b) Let the grammar G = ({S,A}, {a, b}, P, S) where P consists of S →aAS, S → a, A → SbA , A → SS, A → ba. For the string aabbaa find a

(i) leftmost derivation

(ii) rightmost derivation

(iii) derivation tree.

13. (a) (i) Define deterministic finite state automata.

(ii) Draw the state diagram for the deterministic finite state automata, M = $(Q, \Sigma, \delta, q_0, F)$ where Q = $\{q_0, q_1, q_2, q_3\}$, Σ ={a, b}, F = $\{q_0\}$ and δ is defined as follows:

δ	a	b
q_0	q_2	q_1
q_1	q_3	q_0

q_2	q_0	q_3
q_3	q_1	q_2

Check whether the string bbabab is accepted by M. **(3+5)**

(OR)

(b) Given an non-deterministic finite automaton which accepts L. Prove that there exists a deterministic finite automaton that accepts L.

14. (a) (i) Write short on Hasse diagram.

(ii) Let $X = \{2,3,6,12,24,36\}$ and relation $\leq$ be such that $x \leq y$ if *x* divides *y*. Draw the Hasse diagram of $(X, \leq)$. **(4+4)**

(OR)

(b) (i) Show that n^3+2n is divisible by 3 using principle of mathematical induction.

(ii) If the permutations of the elements of {1,2,3,4,5} be given by

$$\alpha = \begin{pmatrix} 1\ 2\ 3\ 4\ 5 \\ 2\ 3\ 1\ 4\ 5 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1\ 2\ 3\ 4\ 5 \\ 1\ 2\ 3\ 5\ 4 \end{pmatrix}, \quad \gamma = \begin{pmatrix} 1\ 2\ 3\ 4\ 5 \\ 5\ 4\ 3\ 1\ 2 \end{pmatrix}, \quad \delta = \begin{pmatrix} 1\ 2\ 3\ 4\ 5 \\ 3\ 2\ 1\ 5\ 4 \end{pmatrix}$$, then find

$\alpha\gamma, \beta\delta, \alpha^{-1}\beta, \gamma^{-1}\beta\delta$. **(4+4)**

15. (a) Prove that there is a one- to-one correspondence between any two left cosets of H in G.

(OR)

(b) (i) If G is a graph in which the degree of every vertex is atlest two, then prove that G contains a cycle.

(ii) Prove that the kernel of a homomorphism g from a group $(G,*)$ to (H,Δ) is a subgroup of $(G,*)$. **(4+4)**

Part C (Answer ANY TWO questions) **2 x 20 = 40**

16.(a) Let G be (p,q)graph, then prove that the following statements are equivalent:

(i) G is a tree. (ii) Every two vertices of G are joined by a unique path (iii) G is connected and $p = q + 1$ (iv) *G* is acyclic and $p = q+1$.

(b) Let H be a subgroup of G. Then prove that any two left cosets of H in G are either identical or have no element in common. **(14+6)**

17. (a) Let $(B,*,(.)',0,1)$ be a Boolean Algebra. Define the operations + and · on the elements of B by,

$a + b = (a * b')((a' * b)$

$a \cdot b = a * b$. Show that $(B, +, \cdot, 1)$ is a boolean ring with identity 1.

(b) Prove that every chain is a distributive lattice. **(15+5)**

18. (a) If G = (N, T, P, S) where N = {S, A,B}, T = {a,b}, and P consists of the following rules: S → aB, S → bA, A → a, A → aS, A → bAA, B →b, B → bS, B → aBB. Then prove the following:

(1) S $\overset{*}{\Rightarrow}$ w iff w consists of an equal number of a's and b's

(2) A $\overset{*}{\Rightarrow}$ w iff w has one more a than it has b's.

(3) B $\overset{*}{\Rightarrow}$ w iff w has one more b than if has a's

(b) State and prove pumping lemma. **(10+10)**

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